

Quark Soup al dente: Applied Superstring Theory

R. C. Myers^{1,2} and S. E. Vázquez¹

¹ Perimeter Institute for Theoretical Physics, 31 Caroline St. N., Waterloo, Ontario N2L 2Y5, Canada

² Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

E-mail: rmyers@perimeterinstitute.ca, svazquez@perimeterinstitute.ca

Abstract. In recent years, experiments have discovered an exotic new state of matter known as the strongly coupled quark-gluon plasma (sQGP). At present, it seems that standard theoretical tools, such as perturbation theory and lattice gauge theory, are poorly suited to understand this new phase. However, recent progress in superstring theory has provided us with a theoretical laboratory for studying very similar systems of strongly interacting hot non-abelian plasmas. This surprising new perspective extracts the fluid properties of the sQGP from physical processes in a black hole spacetime. Hence we may find the answers to difficult particle physics questions about the sQGP from straightforward calculations in classical general relativity.

1. Introduction

Quantum chromodynamics (QCD) is the theory of the “strong force” which determines the physical properties of protons, neutrons and more generally hadrons. At a superficial level, QCD looks like a matrix-valued version of the more familiar electromagnetic theory, replacing photons by gluons and electrons by quarks. However, unlike electromagnetism, quantum fluctuations of the fields play an essential role in determining the force law in QCD. In particular, at low energies or large distances (by the standards of subatomic physics), the coupling of QCD is large and the forces are “strong”, as implied by the original name. This is the origin of confinement, *i.e.*, the fact that we do not see free quarks or gluons in nature but rather we only see “colourless” or QCD-neutral packages known as hadrons. However, at high energies or short distances, the QCD coupling is small and correspondingly the forces are weak. Known as asymptotic freedom, this property allows us to detect the composite structure of hadrons by, *e.g.*, scattering high energy electrons.

Because of the strong coupling, our understanding of many aspects of QCD remains incomplete. For example, while it has now been more than thirty-five years since the discovery of asymptotic freedom, a complete theoretical understanding of confinement remains elusive. In fact, an analytical proof of confinement is now one of the Clay Institute’s “Millennium Problems” for which there is a one-million dollar prize [1]. Of course, great progress has been made on various theoretical fronts. For example, lattice gauge theory [2], which essentially puts QCD on a computer, now provides a fairly good description of the physical properties of low energy hadrons [3]. Another approach is to study QCD away from its ground state. In particular, asymptotic freedom indicates interactions are weaker at short distances and high energies and so one might expect to find new behaviour for QCD at high densities and high temperatures. In fact, a combination of lattice and analytic efforts have allowed theorists to map out the phase diagram of QCD [4] illustrated in figure 1.

Of course, we should also seek an experimental verification of this theoretical picture and this was the goal behind constructing the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory on Long Island [5]. In fact, experiments there have revealed a new phase of hadronic matter called the strongly interacting Quark-Gluon Plasma (sQGP). In this new phase, the quarks and gluons are neither confined nor free but instead, form some kind of strongly interacting “soup” which seems to behave like a near perfect fluid. The sQGP is created at RHIC by colliding two gold nuclei at ~ 200 GeV per nucleon. The collision creates a hot plasma of quarks and gluons which expands outward in a collective flow. The plasma eventually cools down to a temperature where the particles in the plasma are again confined into hadrons which then escape out into the detectors. The red arrows in figure 1 show typical trajectories in the phase diagram which might describe this process. The sQGP would be probed on the upper reaches of these trajectories. Much of this evolution was actually predicted theoretically some time ago in [7], however, a precise interpretation of these experiments now calls for

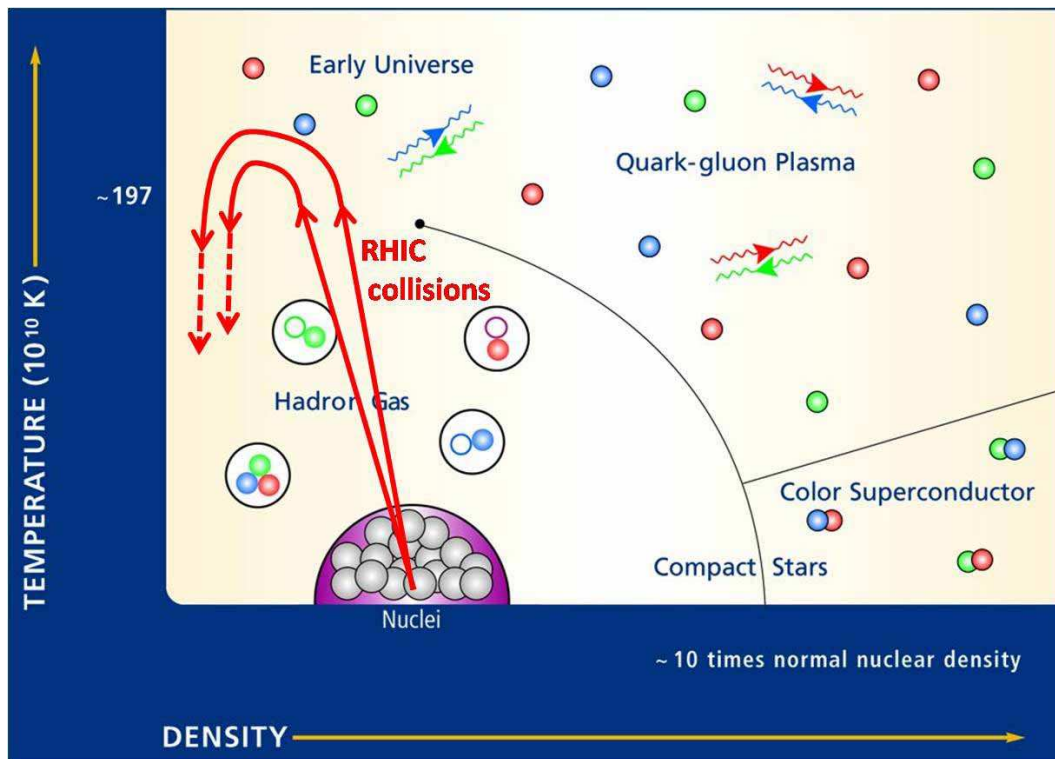


Figure 1. Phase diagram of QCD according to theorists [6]. The red arrows show caricatures of the evolution of matter in typical RHIC collisions.

understanding both strong coupling and also dynamical properties of QCD. Hence we face a challenge since few if any effective theoretical tools exist for this purpose.

At this point, the reader may well be wondering what any of this has to do with gravity, which is the main topic of these proceedings. Quite surprisingly, it turns out a great deal! In a parallel set of developments, string theorists have been uncovering deep connections between strongly coupled gauge theories and gravity. The best understood example of such duality is the so-called AdS/CFT correspondence [8, 9] — for reviews see [10]. These dualities realize a holographic description of quantum gravity in which the theory has an equivalent formulation in terms of a nongravitational theory on the boundary of the original spacetime [11]. A version of this duality involves, on one hand, a (maximally) supersymmetric gauge theory in four dimensions known as $\mathcal{N} = 4$ super-Yang-Mills theory (SYM). The duality proposes that this gauge theory is equivalent to type IIB string theory in an $AdS_5 \times S^5$ background. Further, in a certain strong coupling limit, the latter essentially reduces to classical (super)gravity in this background. In other words, one can calculate gauge theory observables at strong coupling by doing a classical gravity calculations.

It may seem that string theory has provided a way of solving QCD analytically and so we should be ready to collect our million-dollar Millenium Prize. Unfortunately, at present, the gauge theories for which we know their gravity dual are quite different from QCD. Nevertheless, there are regions in phase space for which SYM theory, for example,

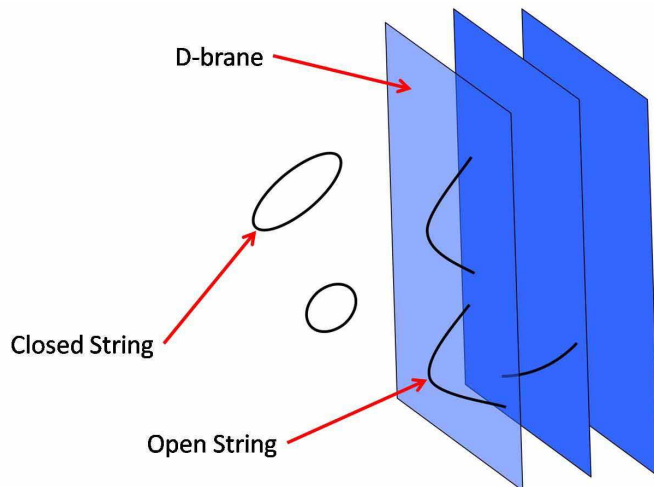


Figure 2. D-branes are solitonic objects of string theory.

seems to share many features in common with QCD. In particular, this happens at high temperatures, precisely in the sQGP phase! The aim of this article is to introduce the AdS/CFT correspondence and explain the relations between QCD and SYM at finite temperature. We will also briefly describe how certain gravity calculations yield observables that are relevant for the physics of the sQGP. Surprisingly, the values calculated using the AdS/CFT duality turn out to be not too far from the ones observed for QCD!

2. The AdS/CFT Correspondence

String theory is much more than just a theory of strings. In particular, it also contains non-perturbative solitonic objects known as Dirichlet-branes or D-branes for short.[‡] Since their discovery in 1995, D-branes have been one of the main stars of string theory [12]. D-branes can be visualized as membrane-like objects where open strings can end (see figure 2).

Of course, string theory has both open and closed fundamental strings, and both can interact with the D-branes. Open strings can be visualized as excitations of the D-brane itself. Closed strings can be absorbed or emitted from the D-brane. Since the closed string modes include the spin-2 graviton, this absorption/emission process is nothing but the backreaction of the D-brane on the spacetime. The string interactions amongst themselves and with D-branes are governed by a parameter known as the string coupling constant g_s . D-branes have a tension of the order $T \sim 1/g_s$, and so at weak string coupling they become “heavy” solitonic objects.

D-branes have different descriptions, depending on where in “parameter space” we

[‡] Sometimes we use the nomenclature Dp -brane, where p is some integer. This integer reflects the number of spatial dimensions in which the D-brane extends. For example, a D-brane with three spatial dimensions is called a D3-brane.

are doing the calculation. For example, if we have some finite number N of D3-branes sitting on top of each other in flat space, one finds that the low energy dynamics of the open string excitations can be described by a non-abelian gauge theory. This theory is precisely four-dimensional $\mathcal{N} = 4$ SYM theory with gauge group $SU(N)$. The two indices of the non-abelian fields, *e.g.*, $(A_\mu)_i^j$ with $i, j = 1, \dots, N$, can be visualized as labeling in which D-brane the open string is ending. Moreover, the string coupling is related to the gauge theory coupling by

$$g_{YM}^2 = 4\pi g_s, \quad (1)$$

where g_{YM} is the SYM coupling. Note that the string coupling g_s is actually related to the expectation value of a field called the dilaton. However, in the string theory solutions that we will consider here, the dilaton takes a constant value and so we can tune g_s to have any value we want. This fact is also related to the conformal symmetry of the SYM theory described above. For a string background dual to a more complicated non-conformal theory such as QCD, g_s would no longer be a constant.

The second description of D-branes is in terms of closed strings. In considering the backreaction of the N coincident D-branes, one finds that the size of their “gravitational footprint” is proportional to $g_s N$. Hence if we consider a strong coupling limit where $g_s N \rightarrow \infty$, we cannot ignore the backreaction on the spacetime geometry. It is well known that the “throat” geometry near the D3-branes takes the form of $AdS_5 \times S^5$ [10]. In the Poincaré patch, the metric is

$$ds^2 = R^2 u^2 \left(-dt^2 + d\vec{x}^2 \right) + R^2 \frac{du^2}{u^2} + R^2 d\Omega_5^2. \quad (2)$$

The five-sphere part of the geometry comes from the six transverse directions to the D3-brane in the ten-dimensional space. The radius of curvature for both the AdS and the sphere is given by,

$$\frac{R^4}{\ell_s^4} = 4\pi g_s N = g_{YM}^2 N. \quad (3)$$

Here ℓ_s is the string scale which is the only free parameter of string theory. It is related to the tension of the fundamental strings by $T_f = 1/(2\pi\ell_s^2)$. In most practical calculations, one performs a Kaluza-Klein reduction on the S^5 and treats the resulting theory as five-dimensional.

In 1998, Maldacena realized that if these two limits, *i.e.*, the low energy and strong coupling limits, were applied sequentially, two radically different pictures emerged depending on the order of limits, as shown in figure 3. However, his bold conjecture was that these two pictures should still describe the same physics [8], realizing a holographic description of the string theory. Hence the AdS/CFT correspondence proposes that the four-dimensional $\mathcal{N} = 4$ $SU(N)$ SYM theory at strong coupling should be equivalent to ten-dimensional superstring theory on the $AdS_5 \times S^5$ background.

This correspondence can be further simplified in a particular regime. First, if we keep the curvature scale less than the string scale, *i.e.*, $R/\ell_s \ll 1$, then stringy corrections to the geometric side of the duality are minimized and we may work with

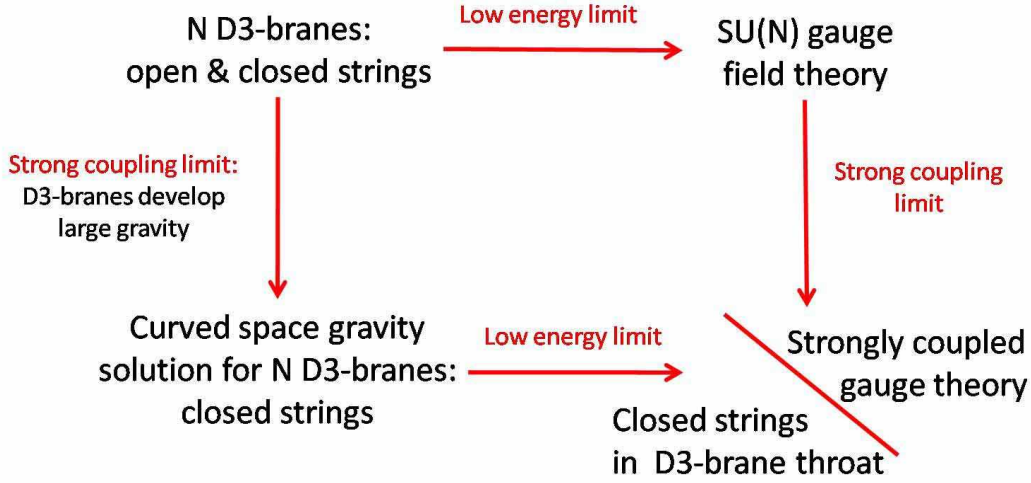


Figure 3. “Derivation” of the AdS/CFT correspondence.

the gravity theory (rather than the full string theory). Similarly if the string coupling is kept small, *i.e.*, $g_s \ll 1$, we also reduce the quantum or loop corrections on the geometric side and so in fact this part of the correspondence is reduced to classical gravity. Given the relation (1), one might worry that the latter limit is trivial for the gauge theory. However, comparing with Eq. (3), we see that maintaining the first inequality above requires

$$N \rightarrow \infty, \quad \lambda \equiv g_{YM}^2 N \gg 1. \quad (4)$$

The limit (4) is also known in the gauge theory as the ’t Hooft limit, where we take the rank of the gauge group N to infinity while keeping λ fixed [13]. In fact, for present purposes, the gauge theory becomes strongly coupled in λ . Hence in the strict large λ limit, the dual theory reduces to just classical supergravity. This is the limit we will be discussing in this article.

The basic dictionary between SYM theory and the gravitational theory was established in [9]. However, many more details of this map have been uncovered during the last ten years. In the most basic set up, the AdS/CFT dictionary relates every field in supergravity (or the full string theory) to a corresponding gauge invariant operator of SYM theory. The purpose of this lecture is not to provide a full review of the subject. For that, we refer the reader to [10]. We will, however, point out the most basic entry in the dictionary, which is the relation between the CFT partition function and the gravitational action.

Given any number of fields in the bulk AdS space (including the metric), the partition function of the CFT is given at strong coupling by,

$$\langle \exp \left(\int \phi_\infty^i \mathcal{O}_i \right) \rangle = \exp \left(-I_{AdS}[\phi^i] \right). \quad (5)$$

The right hand side of (5) is in reality a saddle point approximation when the supergravity approximation is valid. The action I_{AdS} refers to the full bulk action

including the Einstein-Hilbert term evaluated as a functional of all fields that live in AdS . Note that, in the saddle point approximation, this action is evaluated on the solutions to the classical equations of motion for all fields whose boundary condition at infinity ($u \rightarrow \infty$ in (2)) are ϕ_∞^i . One can then calculate all sorts of correlation functions of the gauge theory by functionally differentiating on both sides of (5) with respect to the boundary values ϕ_∞^i .

We will not go into the details of this procedure, which can be quite complicated as the bulk action needs to be renormalized. However, we will point out one result that will be useful later: how to calculate the stress tensor of the CFT. It turns out that the stress energy tensor of the gauge theory is dual to the metric in the bulk. The authors of [14] have given a very general procedure to calculate such stress tensor for any metric in the bulk that is asymptotically AdS .

First, we write the five-dimensional bulk metric in Fefferman-Graham coordinates:

$$ds^2 = R^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} G_{\mu\nu}(x, \rho) dx^\mu dx^\nu \right), \quad (6)$$

where the four-dimensional metric $G_{\mu\nu}$ has the expansion near the boundary (now at $\rho = 0$)

$$G_{\mu\nu}(x, \rho) = g_{\mu\nu}^{(0)} + g_{\mu\nu}^{(2)} \rho + g_{\mu\nu}^{(4)} \rho^2 + \hat{g}_{\mu\nu}^{(4)} \rho^2 \log \rho + \dots \quad (7)$$

The AdS/CFT dictionary in this case tells us that $g^{(0)}$ is the metric on which the CFT is defined (flat space in most cases) and the stress tensor is given by

$$\langle T_{\mu\nu} \rangle_{CFT} = \frac{N^2}{2\pi^2} g_{\mu\nu}^{(4)}. \quad (8)$$

We will make good use of this equation in due course.

A few comments are in order. Note that the spacetime AdS_5 can be seen as the “ground state” of the gauge theory. Any other field that we turn on (including the metric) will result in an excited state of the theory and/or a deformation as in (6). In the example of the stress tensor, the asymptotic metric $g^{(0)}$ is the “deformation”, and the subleading term $g^{(4)}$ will describe in which state the theory is. Given (2), it is easy to show that if our space is exactly AdS_5 with $g^{(0)}$ the Minkowski metric then $g^{(4)} = 0$ and so it follows that $\langle T_{\mu\nu} \rangle = 0$. Of course, this is the correct value for the Poincaré-invariant ground state of SYM. In particular, if we want to study the sQGP, we will need to study the theory at finite temperature. As we will see in the next section, this amounts to replacing the AdS metric with a AdS -black hole in the bulk.

So what do we have here so far? Well, we have uncovered a remarkable new method to study the $\mathcal{N} = 4$ SYM theory in the ’t Hooft limit (4). However, in the introduction, our goal was to study QCD. It seems that SYM is very different from QCD. In QCD, the gauge group is $SU(3)$, *i.e.*, $N = 3$ and we also have $N_f = 3$ flavours of quarks, each of which are fermions transforming in the fundamental representation. § In contrast, for

§ That is, the quarks carry one $SU(3)$ index. Recall from the above discussion, the gluons, which transform in the adjoint representation, carry a pair of indices.

	QCD	$\mathcal{N}=4$ SYM
$T=0$	$N_c=3=N_f$, confinement, discrete spectrum, scattering,	N_c large, N_f/N_c small, deconfined, conformal, supersymmetric,
	very different !!	
$T>T_c$	strongly-coupled plasma of gluons & fundamental matter deconfined, screening, finite corr. lengths, . . .	strongly-coupled plasma of gluons & adjoint and fundamental matter deconfined, screening, finite corr. lengths, . . .
	very similar !!	
$T \gg T_c$	runs to weak coupling	remains strongly-coupled
	very different !!	

Figure 4. Comparison of QCD and $\mathcal{N} = 4$ SYM as a function of temperature.

SYM, we working in the limit of large N and the matter sector contains fermions and scalars, both of which transform in the adjoint representation like the gluons. These extra fields are required to make the SYM theory supersymmetric. However, with this field content, the SYM theory is also a Conformal Field Theory (CFT). That is, it is invariant under conformal transformations, which include the usual scaling of the metric. On the other hand, QCD is not. In fact, QCD is known to produce a dynamical scale which is related to the confining process. Moreover the QCD coupling runs with energy, so the string theory solution (if there is one) will involve a running value of the dilaton which eventually becomes big enough to invalidate the supergravity approximation.

Well, so far the comparison seems hopeless. However, as discussed in the introduction, we are really interested in QCD at finite temperature. The sQGP phase appears just above the critical temperature where the theory becomes deconfining. For the SYM theory, a finite temperature breaks conformal invariance and supersymmetry by introducing a dimensionful parameter in the theory (the temperature). Hence both systems seem to be strongly coupled plasmas of gluons and various matter fields in this regime — see figure 4. Further we emphasize that we would like to model these plasmas with *fluid dynamics*, and so we only care about long wavelength phenomena and not the microscopic dynamics [15]. Therefore, it is very useful to have a strongly coupled gauge theory plasma for which we can do *analytic* calculations. That is, we might hope to gain new insights into the sQGP of QCD from our AdS/CFT calculations for SYM

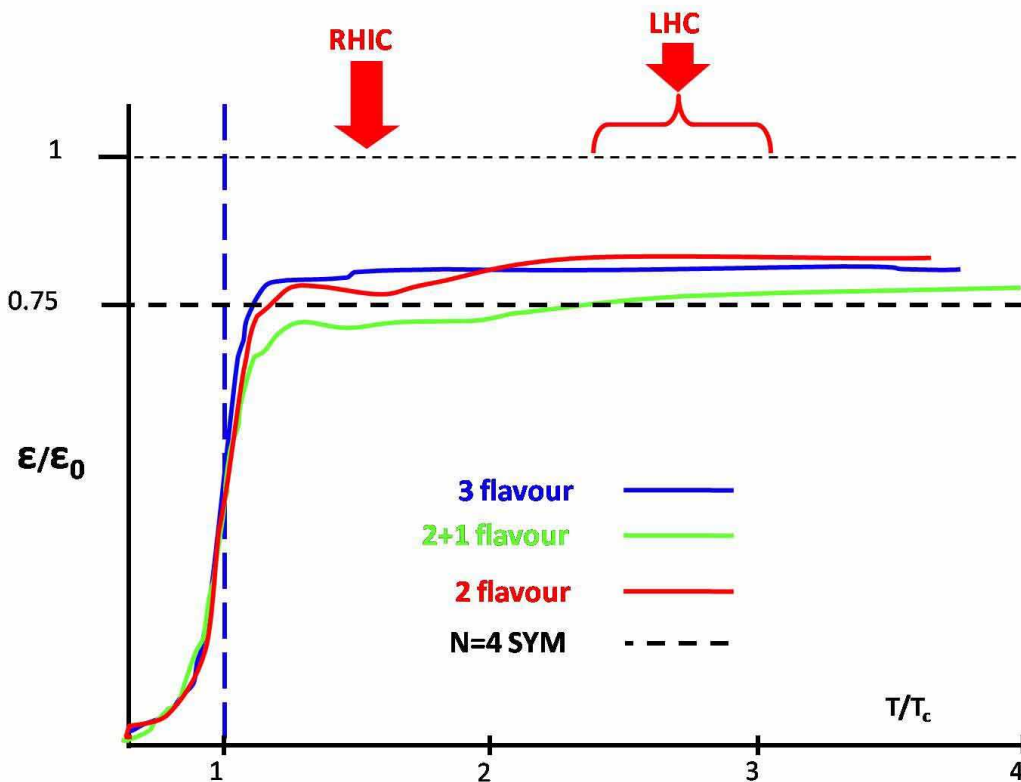


Figure 5. Energy density of QCD and SYM as a function of temperature. The energy density ϵ has been scaled by its value at zero coupling ϵ_0 . The temperature is shown in units of the critical temperature.

at finite T .

While the story above may seem very attractive, one must be wondering if it is more than just hand-waving. In fact, there is some evidence that the similarity between SYM and QCD at finite temperature is more than just of a qualitative nature. One suggestive result is shown in figure 5. This figure shows lattice results for how the energy density ϵ of various theories close to QCD changes with the temperature [16]. However, in this plot, the energy density has been divided by the corresponding density when we turn off the coupling ϵ_0 . We see that when the theories enter the deconfining regime, this ratio rises dramatically and reaches a plateau which is close but still significantly lower than one, *i.e.*, $\epsilon/\epsilon_0 \sim 0.80$ - 0.85 . We might make two observations here: First of all, despite the microscopic differences between the various theories here, they are behaving quantitatively in a very similar fashion when the energy density is presented in this particular fashion. Second, the fact that $\epsilon/\epsilon_0 < 1$ is an indication that on this plateau these theories are still strongly interacting. Now figure 5 also shows the same ratio for the SYM theory. In this case, the ratio is always a fixed constant which takes the value $\epsilon/\epsilon_0 = 0.75$ [17]. Quite remarkably then, despite their microscopic differences, the QCD-like and SYM theories produce quantitative results that are very close to each

other. Hence, this lends concrete support the idea that various aspects of the strongly coupled plasmas may be universal (or nearly universal) and so might be amenable to study with the AdS/CFT techniques.||

In particular, the plateau displayed for the QCD-like theories in figure 5 could indicate that these strongly interacting theories are in a conformal phase where the temperature provides the only relevant scale. Of course, as warned above and in figure 4, the coupling in QCD runs with the energy scale and hence the temperature. Therefore, if we consider QCD at temperatures much higher than T_c , we must find that it behaves as a free gas of quarks and gluons because asymptotic freedom has driven the theory to weak coupling. In contrast, being a conformal theory, the SYM theory will remain at strong coupling for any temperature. Therefore the similarities between QCD and SYM will evaporate again at very high temperatures and it is only in some range of temperatures above T_c where the AdS/CFT correspondence can be expected to provide useful insights. Hence we arrive at the title of our article: to apply AdS/CFT techniques, we should cook the QCD plasma or “quark soup” but not too much!

Our current understanding of the AdS/CFT correspondence allows us to consider gauge theories that are somewhat closer to QCD than SYM. For example, as indicated in the comparison in figure 4, we can add a small number of flavours of “quarks” in the fundamental representation to make a $\mathcal{N} = 2$ supersymmetric theory [20]. The supergravity dual of this theory involves extra D7-branes living in *AdS* background. These extra D-branes can be visualized as the place where the open strings that represent the quarks can end. However, technical issues have restricted these studies to consider $N_f/N \ll 1$. The literature on these constructions is now quite large and so we refer the interested reader to a recent review [21]. One can also find gravity duals to theories that show confinement [22, 23]. Of course, this is all in the ’t Hooft limit and so it is not what we need to understand QCD. Nevertheless, a lot has been learned about the geometrical interpretation of confinement. Further, chiral symmetry breaking has been implemented recently in the popular model of Sakai and Sugimoto [24].

However, our focus here is not on developing a more elaborate correspondence that comes closer to providing a complete description of QCD. Rather, we are interested in studying the known correspondence at finite temperature. In particular, as we mentioned, we are interested in studying the fluid properties of the SYM theory plasma and not it’s detailed microscopic dynamics. In the next section we will explain in more detail how to do finite temperature CFT calculations using just Einstein’s gravity.

3. AdS/CFT at Finite Temperature and the sQGP

How do we calculate properties of the SYM fluid that we can compare with the sQGP? First, we need to learn how to put SYM at finite temperature. For a constant temperature, we will have a static homogeneous plasma. The dual of this phase is given

|| As a word of caution, we should add that the good agreement shown in figure 5 may be somewhat fortuitous since corrections can be expected for both the lattice [18] and the SYM results [19].

by the so-called black three-brane background, which is simply a black hole embedded in AdS_5 [22]. The five-dimensional metric may be written as

$$ds^2 = \frac{(\pi T)^2}{u} \left[-f(u)dt^2 + d\vec{x}^2 \right] + \frac{du^2}{4u^2 f(u)} , \quad (9)$$

where $f(u) = 1 - u^4$ and T is the Hawking temperature. In these coordinates, the horizon is located at $u = 1$ and the asymptotic boundary at $u = 0$.

The calculation of hydrodynamic transport coefficients using this background was pioneered by [25, 26]. A useful review of these results and the subsequent developments appears in [27]. In the original calculations, the transport coefficients, such as the shear viscosity, were determined from correlation functions of the stress tensor using the so-called Kubo formulae. These correlation functions were calculated using the AdS/CFT dictionary. Here we will follow a slightly different but equivalent approach, which we think is less technical. We will directly calculate the stress tensor and will compare it with the expectation from hydrodynamics.

Let us do this first for the black hole background, Eq. (9). We can put this metric in the Fefferman-Graham form (6) by making the change of variables: $u = 2\rho/(1 + \rho^2)$. We can then expand at small ρ as explained in section 2. Reading off the asymptotic coefficients as in Eq. (7), we can easily calculate the stress tensor using Eq. (8). We leave the details for the interested reader. The result is,

$$\langle T_{\mu\nu} \rangle_{SYM} = \frac{\pi^2 N^2 T^4}{8} \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \quad (10)$$

This is precisely the expected stress tensor for a conformal fluid at equilibrium with energy density $\varepsilon = 3p = \frac{3\pi^2}{8} N^2 T^4$. Here the factor T^4 reflects the usual Stefan-Boltzman law while N^2 reflects the number of degrees of freedom in the plasma. Further note that since the energy and pressure densities are related as $\varepsilon = 3p$, $T^\mu{}_\mu = 0$ as expected for a conformal field theory.

Now let us take the plasma a bit out of equilibrium. For a (conformal) fluid out of equilibrium, we can write down the most general stress tensor as,

$$T_{\mu\nu} = \varepsilon u_\mu u_\nu + p \Delta_{\mu\nu} + \Pi_{\mu\nu} , \quad (11)$$

where u^μ is the fluid 4-velocity and the projector $\Delta_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ satisfies $\Delta_{\mu\nu} u^\nu = 0$. The shear tensor $\Pi_{\mu\nu}$ is also orthogonal to the 4-velocity, *i.e.*, $\Pi_{\mu\nu} u^\nu = 0$, as well as traceless $\Pi^\mu{}_\mu = 0$. Moreover, it vanishes at equilibrium.

Now, for a general fluid, the conservation equation

$$\nabla_\mu T^{\mu\nu} = 0 , \quad (12)$$

only determines the energy density and four-velocity of the fluid. The shear tensor remains undetermined and it is the goal of fluid theory to provide extra equations to

determine this quantity. One usually writes such equation as an expansion in derivatives of hydrodynamical quantities. To lowest order, we have

$$\Pi_{\mu\nu} = -2\eta\sigma_{\mu\nu} + \dots, \quad (13)$$

where the constant η is the shear-viscosity and

$$\sigma_{\mu\nu} = \Delta_\mu^\alpha \Delta_\nu^\beta \nabla_{(\mu} u_{\nu)} - \frac{1}{3} \Delta_{\mu\nu} \nabla_\alpha u^\alpha. \quad (14)$$

Given this framework from fluid mechanics, our goal now is to calculate the shear viscosity using the AdS/CFT correspondence. To perturb the fluid, we create a linearized metric perturbation in the bulk. That is, we will solve the linearized Einstein's equations in five dimensions (with a negative cosmological constant) for a metric perturbation of the form: $\delta g_{xy} := (\pi T)^2 h_{xy}(u, x)/u$. This bulk perturbation will also induce a perturbation in the metric on the boundary given by $h_{xy}(0, x) := h_{xy}(x)$.

Now, we have written the fluid equations above for an arbitrary metric. Therefore, we can expand them around the static plasma in flat space. It is an easy exercise to show, using Eq. (13), that the shear tensor is modified, to linear order in the metric fluctuation, by $\Pi_{xy} = -\eta \partial_t h_{xy}(x) + \dots$, where the dots denote higher derivative terms. That is, we are using a metric fluctuation in the boundary to “shake” the fluid.

In practice, one goes to Fourier space and writes

$$\tilde{\Pi}_{xy}(\omega, \vec{k}) = \tilde{h}_{xy}(\omega, \vec{k}) \left[i\eta\omega + \mathcal{O}(\omega^2, \vec{k}^2) \right] \quad (15)$$

We can do the same in the bulk and solve Einstein's equation for each Fourier component $\tilde{h}_{xy}(\omega, \vec{k}; u)$ with $\tilde{h}_{xy}(\omega, \vec{k}; u=0) = \tilde{h}_{xy}(\omega, \vec{k})$. When doing this, one needs to impose some boundary conditions at the horizon $u=1$. These were discussed in [26, 28]. The first is regularity at the horizon (ensuring that we stay within the linearized approximation). The second is that each Fourier mode should have the form of an *incoming* wave towards the horizon. That is, we do not want any fluctuations coming out of the horizon. In [26], this last condition was related to a choice of *retarded* Green's function for the gauge theory.

To lowest order in the frequency ω and setting $\vec{k}=0$ for simplicity, one can easily show that the solution has the form:

$$\tilde{h}_{xy}(\omega, u) = \tilde{h}_{xy}(\omega)(1-u)^{-i\omega/(2\pi T)} \left[1 + \frac{i\omega}{4\pi T} \log(1+u) + \mathcal{O}(\omega^2) \right]. \quad (16)$$

Expanding the full metric (with the fluctuation) around the asymptotic boundary, we can read off the response of the stress tensor using Eq. (8). We get,

$$\tilde{\Pi}_{xy}(\omega) = \tilde{h}_{xy}(\omega) \left[i\omega \left(\frac{\pi N^2 T^3}{8} \right) + \mathcal{O}(\omega^2) \right] \quad (17)$$

Hence comparing the results (15) and (17) we find the value of the shear viscosity of the $\mathcal{N}=4$ SYM plasma to be

$$\eta = \frac{\pi}{8} N^2 T^3. \quad (18)$$

Another important result derived from this calculation is the value of the ratio of the viscosity to the entropy density. Using the thermodynamic relation $s = 4\varepsilon/(3T)$ (at zero chemical potential), we have

$$\eta/s = 1/4\pi . \quad (19)$$

This last result has been the focus of much attention in the past years. It turns out that one can prove that the equality (19) is *universal* for all gauge theories with a gravity dual, in the limit where the gravity approximation is valid (which is usually in the 't Hooft limit) [29, 30]. In fact, it has been argued that *any* fluid that can be obtained from a quantum field theory, will obey the bound [29]:

$$\eta/s \geq 1/4\pi . \quad (20)$$

This is indeed a very strong statement. Some support for this bound comes from calculations which indicate that the leading corrections in $1/\lambda$ to the SYM result (19) in fact increase the ratio so that the bound is no longer saturated [19]. Certainly in nature, no laboratory tests to date have found a substance which violates this bound. However, this conjectured bound (20) has drawn particular attention by preliminary investigations of the experimental data from RHIC which indicated that the sQGP had an unusually low viscosity [31]. Determining a precise value of the viscosity in the sQGP is now an topic of intense study but recent investigations seem to indicate that $\eta/s \sim 1/4\pi$ — for example, see [32].

These unexpected results for the viscosity combined with a lack of alternative theoretical tools to study the strongly coupled dynamics of the sQGP have stimulated tremendous activity in calculating different thermal properties of strongly coupled non-abelian plasmas for the $\mathcal{N} = 4$ SYM and other holographic theories. We are not able to cover all of the literature on this subject but in the following we point the interested reader towards some of the interesting developments.

Above, we showed how to compute the shear viscosity but this only determines the lowest derivative component of the shear tensor $\Pi_{\mu\nu}$, as shown in Eq. (13). In principle, there are an infinite series of higher order terms, each one characterized by a different transport coefficient. At present, there is no complete theory of relativistic fluid dynamics that predicts the form of all such terms. The “standard” theories are of second order in derivatives [33]. In principle one can extend the holographic calculations to higher orders in frequencies and wave-vectors, to uncover the rest of the terms. This procedure has recently been carried out to second order in derivatives in [34]. Identifying the correct set of terms will be crucially important in refining the analysis of the experimental data to determine the precise value of η/s for the sQGP. There have also been calculations of transport coefficients in non-conformal field theories with gravity duals [35]. Of course there have also been a variety of other holographic calculations including: studying the effects on introducing a finite chemical potential [36], investigating spectral functions [37], examining the diffusion of heavy quarks [38] and calculating the “photon” emission rate of the plasma [39].

To conclude then, theorists face many new challenges in developing a physical understanding of the recently discovered strongly coupled quark-gluon plasma. In parallel developments, however, string theorists have developed the AdS/CFT correspondence as a new analytic tool to study certain gauge theories, *e.g.*, $\mathcal{N} = 4$ SYM, which are well suited for this purpose. In particular, in the strongly coupled regime, these gauge theories are dual to a (super)gravity theory in higher dimensions. Further, although these gauge theories differ from QCD in many details, at finite temperature, they seem to share many features in common with the sQGP. Hence gravitational calculations are being used to gain insight into this new phase of QCD as the implications of this remarkable correspondence continue to be explored. At the same time, experimentalists will soon begin to explore a new frontier with heavy ion collisions at ~ 5 TeV/nucleon at the Large Hadron Collider — see figure 5. Thus we can expect to see new surprises coming from both theory and experiment in the near future.

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